CB Asset Swaps and CB Options: Structure and Pricing

San-Lin Chung,
Department of Finance, National Taiwan University and
Department of Finance, National Central University
E-mail:chungs@management.ntu.edu.tw

Hsiao-Wei Lai,
Price Risk Management, Citibank

Shu-Ying Lin[1]
Department of Finance, National Central University and
Department of Information Management, Ta-Hwa Technical University
E-mail: s8445003@cc.ncu.edu.tw

Gang Shyy
Department of Finance, National Central University
E-mail:gshyy@cc.ncu.edu.tw

Abstract

This paper investigates deal the structure of the stripping of convertible bonds into the credit component and equity component, i.e., the CB asset swap and CB option. We provide pricing models for both the credit component and option component for CB Stripping structured products. We show that the CB asset swap can be priced as an American installment option. Our results indicate that a higher asset swap spread paid by the dealer could lead to early exercising of the CB option. The CB option is an American call option with time-varying strike price which is adjusted by the mark-to-market value of IRS. Our simulation concludes that the CB call option will be mostly affected by (1) issuer credit, (2) put price and (3) interest rate level.

Keywords: Asset Swap Transaction, CB Asset Swap, CB Option
JEL: G12

---

[1] Correspondence: Shu-Ying Lin, Department of Finance, National Central University, Jung-Li, 320, Taiwan. Tel: 886-3-422-7151 (ext 6259); fax: 886-3-425-2961; e-mail: s8445003@cc.ncu.edu.tw.
The authors wish to thank Eric Chung at Citibank, H.P. Ueng at China Trust Bank and Irvine Tung at Fubon Securities for information and comments. We also would like to thank two anonymous referees for their valuation comments and suggestions. The work was supported in part by the MOE program for promoting academic excellence of universities under the Grant number 91-H-FA08-1-4.

1. Introduction

Convertible bonds (CB) have become a popular investment tool in recent years, because of their fixed income floor and rich equity option value. In fact, CB issuance was the major growth area in the European capital markets during 2001. Almost 40% of total issuance in the equity and equity-linked markets was via equity-linked instruments (see Euro Money, the guide to global fixed income 2002). The numbers show that the CB market has indeed become a significant focus for issuers and investors.

A CB provides an alternative funding channel for enterprises compared to traditional bonds. Recently, investment banks have developed a sophisticated technique to strip a CB into its credit component and option component in order to arbitrage the preferences between different investor groups. The credit component is commonly referred as a CB asset swap transaction and the option component is referred to as a call on a CB or a CB option. Those dealers who create and market swaps involving CBs benefit from judicious pricing due to high stock volatility, and a swap house can even end up owning potentially valuable equity options at negligible cost. In addition, most commercial banks and insurance companies can demand a higher credit premium on the asset swap side and share the low premium benefit from the CB option holders.

In a CB stripping transaction, the CB is stripped into two structured products: the asset swap (credit component) for fixed income investors and the CB option (equity component) for common equity investors. While a bond investor generally receives an extra spread over the benchmark rate (i.e., Libor or Treasury), an equity investor pays an option premium to have a CB option. To avoid position risk, a dealer sometimes simultaneously matches the asset swap trade with the CB option trade. In other words, the dealer will exercise the right to call the CB and cancel an asset swap transaction when an option investor exercises the CB call option. The equity component and credit component should cancel each other and leave the dealer with an arbitrage profit. Figure 1 shows the structure for a CB stripping transaction for both option and credit components.

The aim of this paper is to determine how many spreads charged in CB asset swap transactions and the value of the CB option. We take the CB as our underlying asset in the valuation of two structured products. The valuation of CBs first appeared in the late 1970s. Ingersoll (1977), and Brennan and Schwartz (1977) both look at CB like a bond plus a warrant. Their pricing model depends on one underlying variable: the firm value. Brennan and Schwartz (1980) extend their previous work and present a two-factor model that includes both stock and interest rate risks. Recently, credit risk is taken into account in the CB pricing model. In

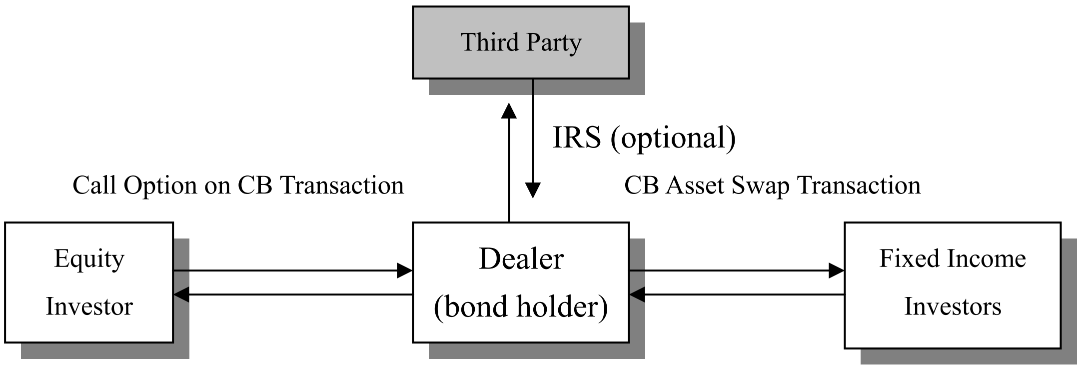


Figure 1 The Structure of CB Asset Swap Business

CB Asset Swap transactions mainly consist of CB asset swap transactions and call options on CB transactions. Third parties may also be involved in these structures for IRS (Interest Rate Swap).

the literature, there are two approaches to defaultable bond pricing: the structural approach (for example, Hull and White (1995), and Longstaff and Schwartz (1995)) and the reduced-form approach (for example, Jarrow and Turnbull (1995); Jarrow, Lando, and Turnbull (1997); and Duffie and Singleton (1999)).

The asset swap is a form of repackaging of a security by using swaps. A par asset swap typically combines the sale of an asset at par with an interest rate swap. The fixed-rate coupon on the bond is paid in return for floating-rate plus a spread (the asset swap spread). The pricing of an asset swap determines the spread, and the pricing techniques usually use the no-arbitrage principle. Chance and Rich (1998) use arbitrage-free replicating portfolios to derive valuation formulas for a variety of equity swaps. For CB stripping transactions, practitioners have come out with different calculation procedures and pricing models for an asset swap and CB option (see Bloomberg (1998)). A common pricing methodology used by them can be summarized as an ad hoc two-step method: First, the equity-option premium is estimated and the fixed income value shows the effective price after subtracting the equity option premium from the CB's current market price. Next, one can use an asset swap calculation procedure to estimate the swap spread that can be achieved with the equity option stripped.

The problem here is that those commercial models ignore the American call feature of a CB asset swap. The extra spread allocated at every coupon date, rather than up-front fees paid in whole, is like the feature of installment products. The right to cancel the swap at any time before maturity resembles the early exercise right of American options. Thus, in our view, the credit component of a CB asset swap can be treated as an American installment option so as to take into account the right to cancel the swap and stop paying future interest payments prior to the ultimate maturity.

Davis, Schachermayer, and Tompkins (2001) introduce the pricing model of European installment call options. Our pricing model is more complex, considering the early exercise right of installment options.

The evaluation model of CB options is similar to that of American call options with time-varying strike price which is a function of three state variables, the short rate, the equity price and credit risk of the CB issuer. The strike price varies with time because it is adjusted by a term called reference hedge, which is the mark-to-market value of the Interest Rate Swap (IRS). Since the reduced-form approach is widely used in practice and has been successfully used in the valuation of credit derivatives (see Sundaresan, 2000), we follow the Jarrow, Lando, and Turnbull (1997) model to deal with a CB subject to credit risk.

The paper is organized as follows: We first introduce a complete CB stripping structure and corresponding transaction contracts in Section 2. Section 3 shows the valuation for a CB asset swap using the concept of an American installment option. The evaluation procedure of a CB option and sensitivity analysis is given in section 4. Finally, section 5 concludes the paper.

2. CB Stripping Structure: TCC CB

As an example, we show the term sheets and framework of a CB asset swap transaction and call option transaction on a CB issued by Taiwan Cellular Corp (TCC) in October 2001. TCC's CB stripping structure is shown in Table 1:

Table 1 Term Sheet of CB Asset Swap

Contract Terms of Initial Bond Purchase: (Seller: Party A, Buyer: Party B)

Bond	Zero Coupon rate, Convertible Bond
Maturity	6 years
Trade Date	October [ ],2001
Termination Date	October [ ],2007
Notional Amount	$ 100
Purchase Price	100 % flat of accrued Interest ($100)
Yield to Put	15 % of notional amount on October [ ],2004
Conversion Price	$ 50 per stock

Contract Terms of CB Asset Swap Transaction

Transaction Maturity	3 years
Trade Date	October [ ],2001
Termination Date	October [ ],2004
Floating Rate Payment (Party A)	Semi-yearly, 6-months LIBOR + spread as floating rate option in accordance with notional amount.
Designated Maturity	6 months
Spread	200 bps

Fixed Rate Payment (Party B)	15 % (Yield to Put) flat of the Notional on termination date
Early Termination	If call option investor exercises his call option, Party A must buy CB back with 100% of notional amount from Party B. And the asset swap will be terminated early.
Exercise Date	Any Business Day up to and including Expiration Date.

Contract Terms of Call Option on CB Transaction

Call type	American call
Underlying Asset	The bonds, as described above
Transaction Maturity	3 years
Seller	Party A (Dealer)
Buyer	Equity investor
Trade Date	October [ ],2001
Expiration Date	October [ ],2004
Premium	10 % flat of national amount
Strike Price	The sum of (1) 100% of the National Amount (2) The net present value of the REFERENCE HEDGE (3) Accrued Interest (4) Early Exercise Fee
Reference Hedge	The differences of future cash flows from the exchange of interest rate payments between the asset swap parties

There are three parts for a CB asset swap transaction:

(1) Outright Sales of CB: A CB stripping transaction begins with an outright sale of a CB from a dealer to a credit investor with a call on the CB attached. While there are various ways to determine the selling price, it is common practice to sell the CB at par value.

(2) Interest Rate Swap (IRS): The IRS refers to a cash flow exchange between the dealer and the credit investor. Most likely, the dealer will pay Libor plus an agreed-upon spread to the credit investor in exchange for a fixed payment of the CB coupon, if any, and a put yield at a put day. Again, market practices differ, but the most common term is that the swap will be terminated if the dealer calls the CB back.

(3) Call Option on CB: The structure of TCC's CB option transaction is shown in Figure 2. Note that the strike price sometimes is adjusted by a term called reference hedge. A reference hedge is the mark-to-market value of the IRS as mentioned above.

In the case of a CB with a high put yield, the CB call option is similar to a default swap mainly to protect the investor from the credit risk. If there is no default risk, then the issuer should redeem the CB at the put price, which is higher than the call price. As a result, the CB option should be similar to a CB holding position.

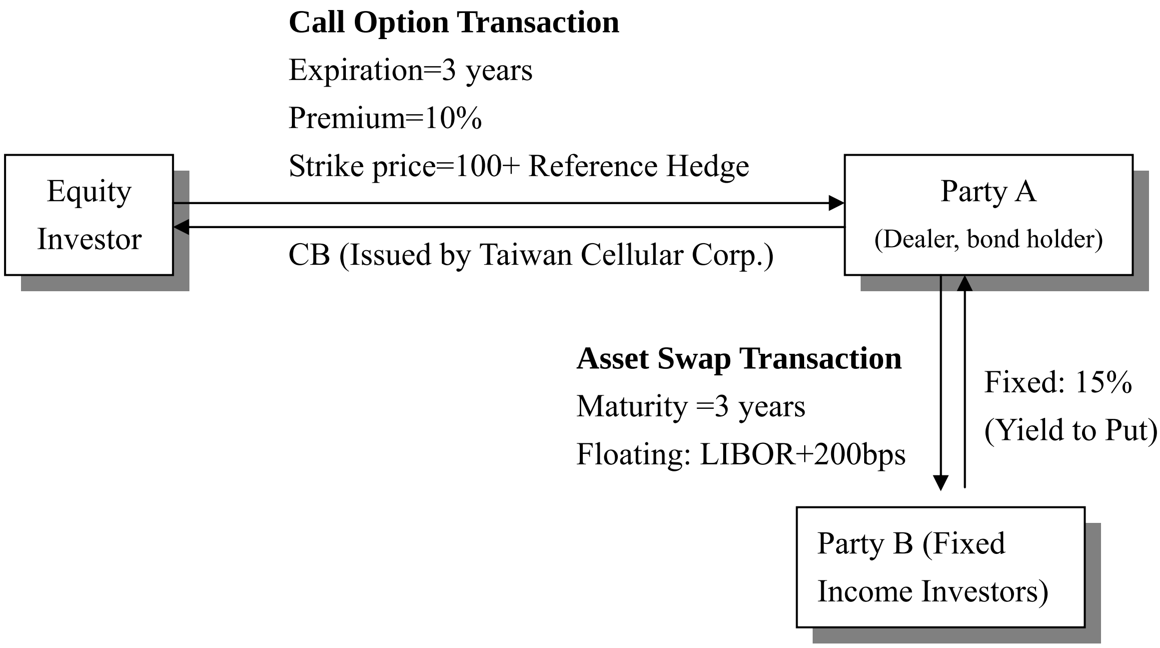


Figure 2 Asset Swap Business Structure of Taiwan Cellular Corp.

However, if the CB issuer goes bankrupt, then the option holder will not call the CB and will protect the investment from the down-side risk.

3. Valuation Framework for a CB Asset Swap Transaction

Recent researchers evaluate derivatives by means of numerical approaches because of the complexity of new financial derivatives without simple closed-form solutions. Brennan and Schwartz (1977) use the finite-difference method to price the CB. Hung and Wang (2001) provide a tree model to price CBs which are subject to default risk.

This section provides detailed valuation procedures to determine the spread of a CB asset swap transaction. The numerical technique of the pricing model here is done by the lattice approach in order to highlight the installment option value and because it is easier to demonstrate. Following this section, we first describe the assumptions in the pricing model, and then our pricing model, American Installment Call (AIC) option valuation procedures, for a CB asset swap transaction. A simple example will be given in section 3.3 to illustrate the pricing procedure of an ad hoc two-step method and an AIC method. Section 3.4 provides Greek letters commonly used for risk management, and section 3.5 concludes the discussion.

3.1. Assumptions

The Cox, Ross, and Rubinstein (1979) multiplicative binomial model showed that options can be valued by discounting their terminal expected value in a world of risk neutrality. We assume here that the market can trade at discrete times. At each interval, the stock can move up or down, and the interest rate can rise, remain unchanged, or decline. After one period, the two-dimension lattice has 6 node points.

The process of the short-term riskless rate r at time t is assumed to follow the CIR (1985) model given by

$$dr = \alpha(\mu_r - r)dt + \sigma_r \sqrt{r} dZ_r, \tag{1}$$

where μ_r is the long-term mean of the short rate, α is the mean-reverting spread, σ_r is the instantaneous standard derivation of the short rate, and dZ_r is the standard Wiener process.

The value of the underlying stock (S) of the issuing firm follows the lognormal diffusion process and the ln S follows a generalized Wiener process:

$$d \ln S = (\mu_s - \frac{\sigma_s^2}{2})dt + \sigma_s \rho dz_r + \sigma_s \sqrt{1-\rho^2} dz_s, \tag{2}$$

where μ_s and σ_s are the long-term mean and the standard derivation of the stock price, respectively. Term ρ is the correlation coefficient between changes in the firm value and changes in the short rate.[2]

3.2 Valuation Procedures for a CB Asset Swap Transaction

CBs are hybrid securities which have the features of a vanilla corporate bond and equity. In a CB asset swap transaction, a dealer sells CB (the part of a vanilla corporate bond) to fixed-income investors and receives par value of bonds as well as a call option written on the CB. The premium of the CB option is allocated at every interest payment date by the spread over Libor. If the CB call option is exercised early, before maturity, there is no more interest payment and the asset swap transaction is terminated.

Similar to an installment option, the periodical interest payments (Libor plus spread) can be treated as an installment for the option premium of a CB option. Most importantly, a CB asset swap can be regarded as an American installment option, because the dealer can call the CB and terminate the asset swap transaction before maturity. As a result, an early call on a CB could reduce the future interest payments (i.e., option premium installment) significantly. To incorporate the installment payment structure into the pricing model, we conduct our calculation procedure in the following manner.

The fixed income investor on the settlement date pays the bond dealer the full

[2] Since there is no common accepted tree building method for a credit spread, credit risk is not considered here, but will be taken into account in the pricing call option next section.

face value plus accrued interest for the bond. It is important to note that the difference between the par and the market value of a CB is the up-front premium payment in our model. A fair valuation on the CB asset swap transaction is to determine a spread over Libor in order to equate the theoretical up-front premium with the market-par difference. A recursive procedure is required to determine the spread.

The valuation procedure is a standard backward recursive pricing method. If the call option on a CB is held until maturity, then the payoff should be the value of the CB at that time after deducting the strike price. The payoff of the installment option at maturity date is given as

$$\text{Max (CB Price – Strike Price, 0).} \tag{3}$$

Proceeding one step forward, on each decision node the option holder makes a decision based on the following considerations: First, the dealer has the right to decide whether to keep the call or to exercise it early. If the decision is to keep the option, then the dealer has to pay Libor plus a spread for one more period. In other words, the interest payment is treated as a sequential premium installment of the CB option. If dealer decides to call the CB back, then the asset swap is terminated automatically and no more future payment is required. Thus, the holding value of a CB option needs to be further deducted by a one-period, discounted floating payment. The payoff of the installment option before maturity date can be expressed by

$$\text{Max (CB Price – Strike Price, Holding Value – Floating Payment).} \tag{4}$$

The holding value of a CB option is the discounted future cash flow of the next period. The payoff of each node is obtained by repeating the procedure described above. The floating payment is the sum of Libor and spread charged. The spread is determined when the call value at contract day is equal to the difference between the CB's theoretical value and the initial exchange amount.

3.3 An Example

We apply the AIC method to price the spread in a CB asset swap transaction by a simple numerical example. Considering one unit of a zero-coupon CB with par value 100, it can be converted to two shares of stock at a conversion price of 50. The put price of the CB at the end of the period is 15% of the notional amount. The call option on the CB can be exercised monthly at a strike price of 100. The expiration date of the call option is three years.

Ad Hoc Two-Step Method

We now turn to decide the spread set in a CB asset swap transaction according to the ad hoc two-step method. The first step is to decide the effective price after taking the equity option premium away from the current CB market price. As shown in Table 2, the effective price is 95.06 or 142.07-47.01.

Table 2 Summary of Pricing Results

The spread is determined by two methods in the following way: ad hoc Two-Step Method: Spread is adding up to floating payment of IRS to equate the mark-to-market value of IRS equal to the Effective Price-Par difference. Effective price is defined as CB value minus the value of Call on Equity. AIC Method: The spread is set to equate the up-front premium with the market-Par difference.

The results are obtained by the following parameters: the initial interest rate 5%, mean-reverting spread 40% per year, long-run mean of interest rate 5% per year, volatility of interest rate 10% per year, initial stock price 50, long-run mean of stock price 5% per year, volatility of stock price 50% per year, correlation coefficient 30% per year. All the parameters chosen are for illustrative purposes only and do not necessarily correspond to the economic environment.

	Perfect Market	Illiquid CB Market
Ad Hoc Two-Step Method		
CB Value	142.07	138.65
Value of Call on Equity	47.01	47.01
Effective Price	95.06	91.63
IRS Mark to Market on settlement date	-4.94	-8.37
Spread Calculated by Ad Hoc Method	158 bps	281 bps
AIC (American Installment Call) Method		
Value of AIC with Zero Spread	42.11	42.11
Spread Set by AIC Method		378 bps

We next use an asset swap calculation procedure to estimate the swap spread. By the concept of cashflow, at contract day, the cash received and cash paid by the dealer should be equal in the sense of no arbitrage opportunity. That is, the dealer receives par value in cash and gives up the 'pure bond' part decomposed from the CB, which is referred to as the effective value in the CB asset swap. The difference of the par and effective price is the value of IRS at origination.

The effective price is usually less than par. In this case, we have to determine the spread that makes the reference hedge, i.e., the mark-to-market value of the IRS, equal the cash outflows at settlement day. More precisely, the spread is determined by the following equation:

$$\text{Reference Hedge} + \text{Par Value} = \text{CB value} - \text{Call Value, or} \tag{5}$$

$$\begin{aligned}\text{Reference Hedge} &= \text{CB value} - \text{Call Value} - \text{Par Value}\\ &= 142.07 - 47.01 - 100\\ &= -4.94.\end{aligned}$$

The spread of asset swap is set when the mark-to-market value of the IRS is equal to -4.94. In principle, the cash outflows of IRS at each coupon date are as follows: {r_1+spread, r_2+spread, …,r_{n-1}+spread}, where r_i is the short-term riskless rate at time *i,* and time *n* is maturity. IRS receives the difference of put price, or 115, and the par value of bond, or 100, at the end of maturity. The mark-to-market value of IRS at present is computed simply by discounting, applying the short-term riskless rate at each period, whatever the cash outflows, and comparing that present value with the present value of cash inflows. In our example, the net present value of IRS is -4.94 when the spread is increased by 158 bps.

AIC Method

Due to the sure yield-to-put return at maturity, which is usually greater than the strike price, and also due to the possible gain from the stock price rising, the value of the CB is very deep in the money in our example. We refer to a market as perfect if a dealer can sell the CB at a reasonable price or theoretical price. In this situation there is no motivation for a dealer to deal with a CB asset swap transaction. As shown in Table 2, the value of an installment call with zero spread added is 42.11, which is a little bit greater than the difference between the CB and the par value of the CB asset swap contract. The tiny difference means that the time value of the call option is negligible. Therefore, the dealer would prefer to hold the CB by himself.

In the reality, the market price of a CB is less than the theoretical price for some reasons such as trading in an illiquid secondary CB market. We refer to the market as illiquid if the CB holders only have two choices: either hold the underlying asset until maturity or convert the bond into equity immediately. The payoff of CB is as follows:

$$\text{Max (Conversion Value, Holding Value)} \tag{6}$$

The conversion value is the conversion ratio multiplied by stock price at the time of conversion. The holding value in a liquid market is the discounted CB value of the next period, while in an illiquid market, the holding value is the discounted last-period payoff of the CB. Thus, the price of a CB in an illiquid market is less than its price in a liquid market. The illiquid market price of the CB is 138.65 as shown in Table 2, and the spread calculated by the ad hoc two-step method is 281 bps, which is greater than the 158 bps charged in a perfect market. This is because a lower CB price will diminish the effective price and the larger gap between par value and effective price will be covered by higher interest payments on each coupon date.

The results of spread charged are shown in Table 2; the spread of a CB asset swap by the AIC method is 378 bps, which is larger than that by the ad hoc two-step method, or 281 bps. The higher spread of the AIC method is consistent with some market observations: First, we find that most CB asset swaps are called mainly for interest concerns in a declining interest rate environment. The ad hoc approach mainly focuses on the equity option value of a CB asset swap. As a result, a CB Asset Swap should be priced much higher than a simple equity option as defined in the ad hoc method. In addition, the opportunity cost of going into the next payment period (installment cost) also add incentive for early exercising. This observation seems to support our AIC method.

American- v.s. European-Style Options

In order to demonstrate the influence of spread charged on the decision of whether to exercise early, we show the relationship between the spread charged and the theoretical up-front fee for American-style and European-style options in Figure 3. For the American installment option, the chance of exercising the option early increases with a higher periodical option premium. Thus, the asset swap spread can be treated as part of the cost of holding the option. The payoff of the installment option is the larger of conversion value and the net holding value (the difference of holding value and floating payment) as shown in Equation (4). When the spread is too high to hold the option, the dealer will exercise it early, and the payoff of the option is the conversion value, which is independent from the spread charged. As shown in Figure 3, the upper line, the value of AIC option, is almost flat as spread increases.

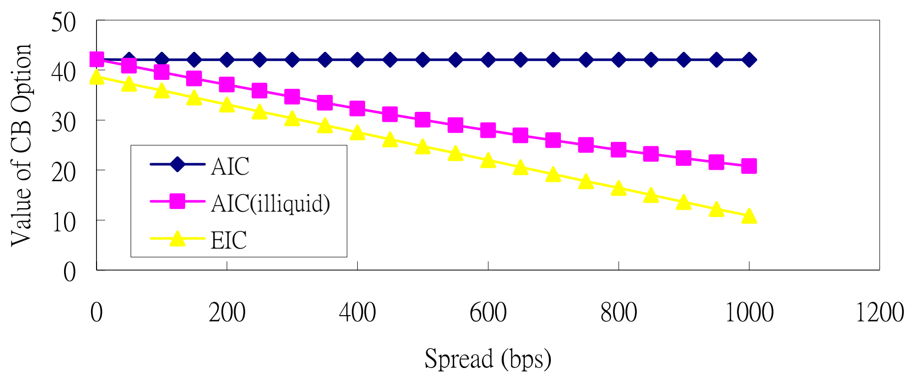


Figure 3 Value comparison for call on CB of American and European options

The upper and lower lines are the values of American installment call option (AIC) and European call option (EIC) on CB at present time in the perfect market. The middle line is an American installment call option on CB in the illiquid secondary CB market (AIC (illiquid)).

For installment options, the value of an American type at the beginning of the swap transactions can be shown to be much higher than the value of a European style, because the dealer has the right to exercise early before maturity and is not required to pay any further premium. We do see a negative relationship between the up-front fee and spread charged, since the spreads are part of the premium to be paid in the future. A higher spread reduces the holding value of an option and this effect is more significant for the European type. It is interesting to note that the gap between the two lines widens as the spread increases. This suggests that a higher asset swap spread will lead dealers to exercise the option early.

The middle line in Figure 3 shows the relationship between the up-front fee of an American installment at the beginning and the spread charged in an illiquid secondary CB market. This line is a downward curve with more spread added, but the slope is decreasing.

3.4 The Greek Letters

Risk management is an important topic and has become a significant focus for investors. In this study we concentrate on what are commonly referred to as the 'Greek letters'. Each Greek letter measures a different dimension of the risk in a specific derivative product position and is defined as the rate of change of the derivative price with respect to one exogenous variable, such as the price or the volatility of an underlying asset. For example, delta is the impact of changes in the price of the underlying asset on the value of the derivative price.

For the ad hoc two-step method and AIC method, the calculations of the Greek letters are distinct. Practitioners strip a CB into three irrelative parts: Outright sales of a CB, IRS, and a call option on a CB. On the settlement date, the outright price is determined and is fixed after then. Neither the stock price nor interest rate level can affect this price. Thus, Greek letters are zero for the part of outright sales. For the IRS, the mark to market value is influenced only by the parameters related to the interest rate. Following the standard procedures, we calculate the Greek letters for each transaction. Adding those three parts together, we have the Greek letters for the ad hoc two-step method as shown in the fourth column of Table 3.

Measured by Greek letters, the risk of the ad hoc two-step method is more than that of the AIC method, especially for the rho factor. In our AIC pricing model, the IRS is imbedded in the installment option. Thus, rho is calculated by the ad hoc two-step method, resulting in 480, which is higher than that by the AIC method, 376, because the IRS can be cancelled out.

Table 3 Greek Letters in Illiquid CB market

In an illiquid CB market, the payoff of CB is max(DLCF, CV), where DLCF is discounted last period cash flow, and CV is the Conversion Value. The Greek Letters of ad hoc Two-Step method are the sum of Greek Letters of CB option transactions and of IRS. Each Greek Letter is defined as follows:

$Delta = \dfrac{c(s+h)-c(s-h)}{2h}$, h: small positive number, c(.): derivative value, s: stock price.

$Gamma = 2*\left[\dfrac{c(s+h)-c(s)}{h} - \dfrac{c(s)-c(s-k)}{k}\right]/(h+k)$, k: small positive number.

$Rho = \dfrac{c(r+h)-c(r-h)}{2h}$, r: interest rate.

$Vega_s = \dfrac{c(\sigma_s+h)-c(\sigma_s-h)}{2h}$, σ_s: the standard error of stock price.

$Vega_r = \dfrac{c(\sigma_r+h)-c(\sigma_r-h)}{2h}$, σ_r: the standard error of interest rate.

Greek Letters	CB Option	IRS	Ad Hoc Method	AIC Method
Delta	1.6211	0	1.6211	1.5803
Gamma	0.0074	0		0.1666
Rho	-343.0901	-137.3997	-480.4898	-375.8812
$Vega_s$	78.5871	0	78.5871	75.5846
$Vega_r$	-205.7667	8.1441	-197.6226	-207.7406

3.5 Discussion

As mentioned before, practitioners use an ad hoc two-step model to determine the level of the spread in a CB asset swap transaction. Since the two-step model does not consider the early call feature, the spread will be under-estimated in a European-style valuation model. However, the magnitude of the mispricing depends on the possibility of the dealer exercising the CB option early and canceling the asset swap. If the underlying CB is deep in-the-money, then the early exercising possibility increases and the American-style spread should be much higher.

On the other hand, if the underlying CB is deep out-of-money or the put price is high, then the dealer is more likely to wait until the expiration day and the American-style spread should be similar to the European-style spread. One way to fix the pricing problem is to adjust the strike price with a term called the reference hedge. In other words, the contract of a CB asset swap sometimes specifies that the strike price for the CB option should be adjusted by the mark-to-market value based on future asset swap payments to maturity. However, due to calculation complexity and the lack of a benchmark for the term structure of the CB asset swap, a dealer tends to use a fixed strike price unless requested by a sophisticated credit investor.

4. The Pricing Model of a CB Option

This section describes the evaluation procedure of a CB option transaction. There are three state variables: the short rate, the equity price and credit risk of the CB issuer. Binomial techniques can be used for pricing American options. However, they become impractical in situations where there are multiple factors. On the other hand, the Monte Carlo method was previously restricted to use in European option valuation because it cannot handle the situation of an early exercise. In recent years, the Monte Carlo simulation approach has been shown to have other advantages as a framework for valuing American options. For example, Longstaff and Schwartz (2001) develop a new least-squares approach to value American options to overcome the shortcomings of the Monte Carlo simulation method.

In this section, we first describe assumptions imposed in pricing model of CB option. We briefly summarize the evaluation procedures of a CB option in section 4.2 and the simulation results in section 4.3. Finally, the sensitivity analysis of various parameters is given in section 4.4.

4.1 Assumptions

The numerical method we use here follows the least-squares approach purposed by Longstaff and Schwartz (2001). The least-squares approach is applied to derivatives' pricing that not only depends on multiple factors, but also has path-dependent and American-exercise features.

The stochastic process of two underlying variables, stock price and the spot interest rate, of the model are the same as in section 3.1. As to the credit risk of a CB, we extend the concepts of intensity models and adopt credit ratings as the classification of different credit spreads. Jarrow, Lando, and Turnbull (1997) propose a model in which the bankruptcy process follows a discrete state space Markov chain in credit ratings. This approach explicitly incorporates a firm's credit rating as an indicator of the likelihood of default and specifies a time-homogeneous finite state space Markov chain with a generator matrix:

$$\Lambda=\begin{pmatrix} \lambda_1 & \lambda_{12} & \cdots & \lambda_{1,k-1} & \lambda_{1k} \\ \lambda_{21} & \lambda_2 & \cdots & \lambda_{2,k-1} & \lambda_{2k} \\ .. & & & & \\ \lambda_{k-1,1} & \lambda_{k-1,2} & .. & \lambda_{k-1,k-1} & \lambda_{k-1,k} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \tag{7}$$

The off-diagonal terms of the generator matrix λ_{ij} represent the transition rates jumping from credit class i to credit class j. The last term of rows λ_{ik} means that the credit class i goes straight to default. For example, a bond rated AAA, the highest rating available, would be in position 1. The transition rate of staying in its class is λ_1, the transition rate of being down-rated by one rating class is λ_2, etc.

The corresponding credit spreads are assumed in Table 4.A. Recently, there have been a few articles discussing the estimation of risk neutral parameters. For example, Ait-Sahalia and Lo (1998) construct a nonparametric estimation for the state-price density implicit in option prices. However, there are too many parameters (42 parameters in our case) required from market data for the estimation. Jarrow, Lando and Turnbull (1997) suggest using a decomposition of the generator matrix under the martingale measure into a product of the empirical generator matrix and a vector of time-dependent risk premium. Since highly rated bonds have low default probabilities within a period of time, this makes the risk premium adjustments ill-defined. Kijima and Komoribayashi (1998) propose new risk premium adjustments to overcome this drawback.

We use the risk premium adjustments based on 1997 data calculated by Kijima and Komoribayashi (1998) and simulate the estimated value of CB according to 1-year transition matrices in the real world and in the risk-neutral world. We find that the simulated values of CB are not very different (139.294 and 139.117 based on the historical and risk-neutral transition matrices, respectively). Thus, it is safe to assume, for relatively short-maturity instruments, that a historical transition matrix can be used to approach a risk-neutral transition matrix for the sake of simulation and comparison.

For the rest of the paper, we use for simulation purpose the average 1-year transition probabilities from 1981 to 2001 given by Standard and Poor's Special Report (2002) as shown in Table 4.B. We adjust these data for the removal of the "not rated" category and classify the credit ratings into seven categories: AAA, AA, A, BBB, BB, B and C (the credit ratings CCC and D in Table 4.B). Considering that the credit rating of CB doesn't change too often in practice, a quarterly transition matrix would be suitable for simulating credit rating paths. We derive the quarterly transition matrix in Table 4.D by the idea of $A^4 = B$, in which A represents a 1-Quarter time-homogeneous generator matrix while B represents a 1-Year transition matrix.[3]

The process for simulating a credit path is as follows:[4] Based on the quarterly transition probability matrix, the probability that an AAA credit rating at the present time stays at AAA one quarter later is 0.9827; the change of becoming AA, A, BBB, BB, B, and C credit rating is 0.0163, 0.0007, 0.0002, 0.0001, 0, and 0 respectively. We sample the simulated transition probability from a uniform distribution over an interval [0,1]. Accounting for the cumulative distribution function of the initial

[3] As pointed out by the reviewer, our simulation results are based on the assumption that the rating changes at each quarter are independent from each other.

[4] The rating process is a multinomial distribution. There are seven rating categories, from the highest rating AAA to the worst rating C, into which the firm will be assigned next quarter.

Table 4 The Assumptions of Credit Spread and Transition Probabilities

Credit spreads for each credit rating are assumed in Table 4.A. The average 1-Year transition probabilities from 1981 to 2001 are given by Standard and Poor's Special Report: Rating Performance 2001 (2002) as shown in Table 4.B. We modify these data into a 1-Year transition matrix by the proportion of transition probability in each rating. For example, the modified probability, shown in Table 4.C, that an AAA-rated bond remains AAA at the end of one year is 0.8962/(1-0.0393), or 0.9329. The quarterly transition matrix shown in Table 4.D is derived by the idea of $A^4 = B$, which A represents 1-Quarter time-homogeneous generator matrix while B represents 1-Year transition matrix as shown in Table 4.C.

4.A: Credit Spread

Credit	AAA	AA	A	BBB	BB	B	C
Spread(bps)	10	50	100	150	300	1000	5000

4.B: Average 1-Year transition probabilities (Rating at the end of year)

Initial	AAA	AA	A	BBB	BB	B	CCC	D	NR*
AAA	0.8962	0.0592	0.0043	0.0009	0.0003	0.0000	0.0000	0.0000	0.0393
AA	0.0060	0.8829	0.0678	0.0051	0.0005	0.0009	0.0002	0.0001	0.0366
A	0.0006	0.0210	0.8779	0.0504	0.0043	0.0017	0.0004	0.0005	0.0433
BBB	0.0003	0.0023	0.0436	0.8443	0.0415	0.0073	0.0023	0.0026	0.0558
BB	0.0002	0.0006	0.0041	0.0575	0.7598	0.0705	0.0109	0.0122	0.0842
B	0.0000	0.0008	0.0027	0.0035	0.0477	0.7415	0.0390	0.0596	0.1053
C	0.0011	0.0000	0.0022	0.0067	0.0145	0.0895	0.5134	0.2472	0.1253

* NR stands for 'not rated'

4.C: 1-Year Modified Transition Probabilities (Rating at the end of year)

Initial Rating	AAA	AA	A	BBB	BB	B	C
AAA	0.9329	0.0616	0.0045	0.0009	0.0003	0.0000	0.0000
AA	0.0062	0.9164	0.0704	0.0053	0.0005	0.0009	0.0003
A	0.0006	0.0220	0.9176	0.0527	0.0045	0.0018	0.0009
BBB	0.0003	0.0024	0.0462	0.8942	0.0440	0.0077	0.0052
BB	0.0002	0.0007	0.0045	0.0628	0.8297	0.0770	0.0252
B	0.0000	0.0009	0.0030	0.0039	0.0533	0.8288	0.1102
C	0.0013	0.0000	0.0025	0.0077	0.0166	0.1023	0.8696

4.D: Quarterly Modified Transition Probabilities (Rating at the end of quarter)

Initial Rating	AAA	AA	A	BBB	BB	B	C
AAA	0.9827	0.0163	0.0007	0.0002	0.0001	0.0000	0.0000
AA	0.0016	0.9782	0.0188	0.0010	0.0001	0.0002	0.0001
A	0.0001	0.0058	0.9783	0.0142	0.0010	0.0004	0.0002
BBB	0.0001	0.0005	0.0124	0.9718	0.0122	0.0017	0.0012
BB	0.0001	0.0001	0.0009	0.0175	0.9535	0.0219	0.0060
B	0.0000	0.0002	0.0008	0.0006	0.0152	0.9522	0.0311
C	0.0003	0.0000	0.0006	0.0020	0.0040	0.0289	0.9642

credit rating, we assign the credit rating of the next period. For example, if the sample is between 0 and 0.9827, the credit rating of an initially AAA-rated firm next quarter is still AAA. If the sample lies in the 0.9827 to 0.999 (0.9827+0.0163) range, then the credit rating of the next quarter is moved from AAA to AA. The rating of the next quarter is A if the sample is in the range of 0.999 and 0.9997 (0.999+0.0007). The credit ratings of A, BBB, BB, B, and C are assigned in the same way described above.[5]

4.2 Pricing Model of a CB Option

We price the estimated value of a CB option by the standard pricing method of American call options, except that the strike price is affected by a term called the reference hedge. It is like a time-varying strike price of an American call if we treat the sum of the contract strike price and reference hedge as the exercise strike price. At maturity, the reference hedge is the net cash flow of the IRS, which is yield to put minus the last period's floating payment. Thus, the payoff of a CB option can be expressed by the following equation

$$\text{Max (CB Price– Strike Price – Reference Hedge, 0).} \quad (8)$$

Before maturity, since this is an American option, the option holder has the right to exercise early, and thus the payoff of the CB option before maturity date is

$$\text{Max (CB Price – Strike Price – Reference Hedge, Continue value).} \quad (9)$$

The continuous value is the conditional expected value of a simple regression in the least-squares approach. There are four state variables, stock price, risk-free interest rate, credit spread, and CB value, which influence the value of a call option on a CB. The multiple linear regression model is used here, and the generic form of this model is

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \varepsilon_i \text{, } i=1,\dots,n, \quad (10)$$

where y_i is the corresponding discounted cash flows of one period further, and x_{i1}, x_{i2}, x_{i3}, and x_{i4} are stock price, risk-free interest rate, credit spread, and CB value, respectively, while ε_i is the error term.

According to Longstaff and Schwartz (2001), we use only in-the-money paths since this allows us to better estimate the conditional expectation function in the region where exercise is relevant and significantly improves the efficiency of the algorithm. In order to reflect the risk of bankruptcy, we use the sum of interest rate and credit spreads in the prior period as our discount rate. The reference hedge is the difference of future cash flows from the exchange of cash flow payments between the

[5] As the referee mentioned, the credit rating of the next quarter depends on the initial credit rating because the transition probabilities or the cumulated probabilities vary with different initial credit ratings.

CB asset swap parties.

Note that when the option is exercised at a specific time, say *X*, the cash flow after time *X* becomes zero. This result accounts for the conversion option can only be exercised once. Proceeding further, we can estimate the value of a call on a CB at the present time.

4.3 Simulation Results

Following the pricing procedure described above, we simulate the value of a CB in the example mentioned in section 3.3. Assuming that the initial credit rating of the CB is A, then the initial credit spread is 1% according to Table 4.A. Based on 50,000 paths for the process of stock price, interest rate, and credit rating with 156 discrimination points during the maturity of 3 years (which indicates there is one exercise opportunity each week), the estimated value of a call on the CB is 40.02 while the estimated value of the CB is 139.33.

Aside from pricing the value of an American call on a CB, we are interested in the differences between the estimated values of an American call on a CB and the estimated value of a European call on a CB using the same pricing approach. In addition, we examine the effect of credit risk. We compare the results in a simulated value of an American option as well as a European option with and without credit risk. The comparison results are shown in Table 5.

Table 5 Value Comparison of European and American Options with and without Credit Risk

The numerical method follows the least-squares approach developed by Longstaff and Schwartz (2001). The simulation results are based on 50,000 paths with 156 discrimination points before the maturity. Initial Credit rating of CB is A.

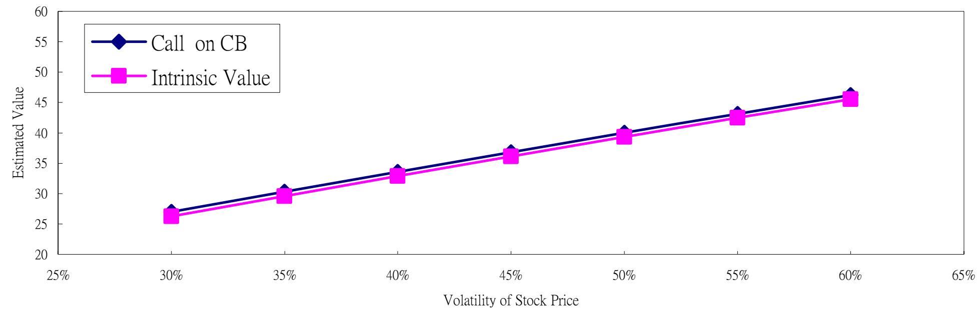
	Without Credit Risk		With Credit Risk	
	European	American	European	American
CB	143.21	143.21	139.33	139.33
Call on CB	39.51	43.59	35.20	40.02

We first compare the estimated values of a European call with an American call in Table 5. The estimated value of an American call is higher than that of a European call, because of the early exercise right. We notice that the CB's estimated value in excess of strike price 100 is higher than the estimated value of a European call. This is because we can convert a CB into stock anytime before maturity, while we can only exercise the European call on a CB on the expiration date.

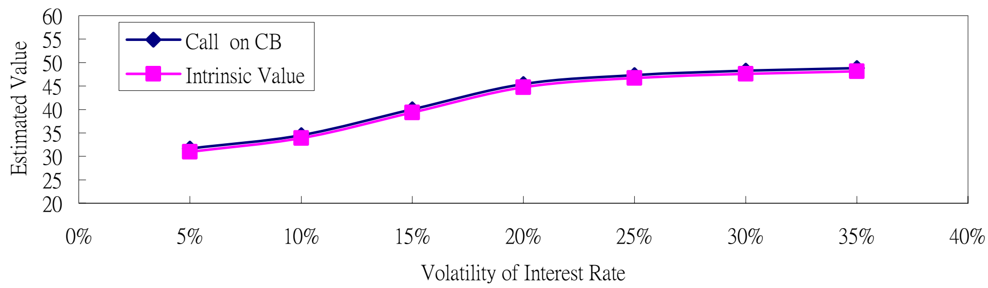
The simulation results considering the credit risk of the underlying CB are shown in the last two columns. Since the initial credit rating of the CB is assumed to be A, the credit spread added into the discount factors is around 1% per year. This results in around a 3% difference in the simulation outcomes when considering credit risk compared to that without considering this variable.

4.4 Sensitivity Analysis

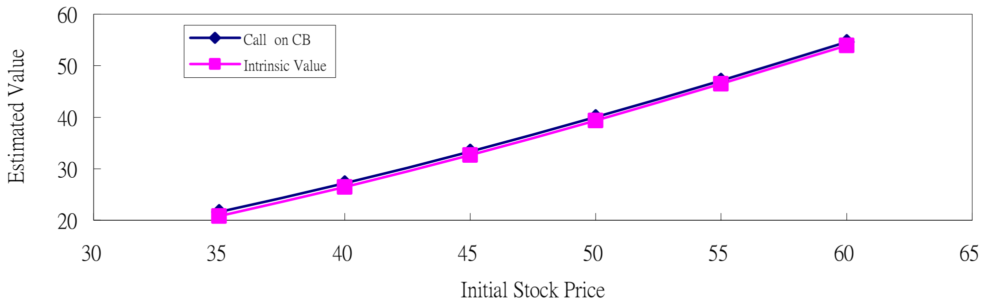
This section investigates the sensitivity of various parameters on the pricing model. The simulation results of a CB option and a CB's value in excess of strike price 100, which we define as the intrinsic value, are shown in Figure 4 for various parameter values.



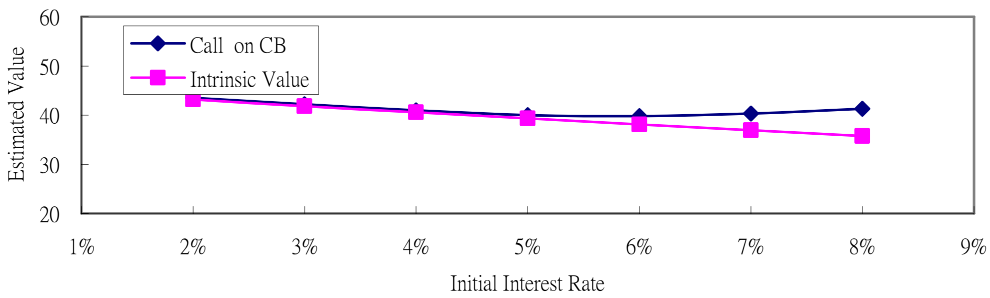
4(a) Effect of Volatility of Stock Price



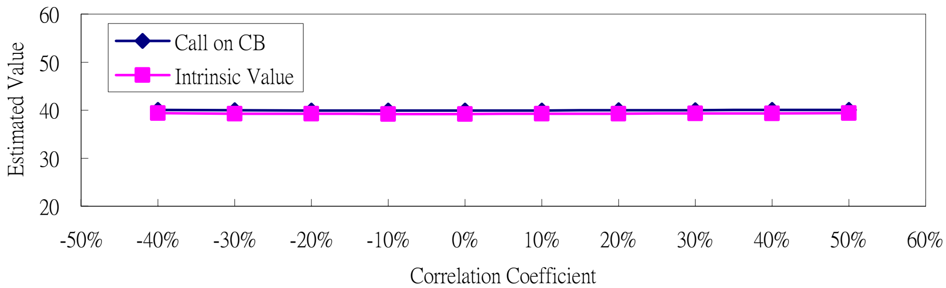
4(b) Effect of Volatility of Interest Rate



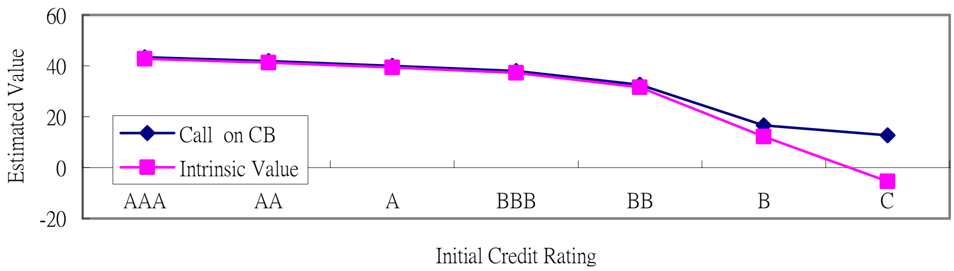
4(c) Effect of Initial Stock Price



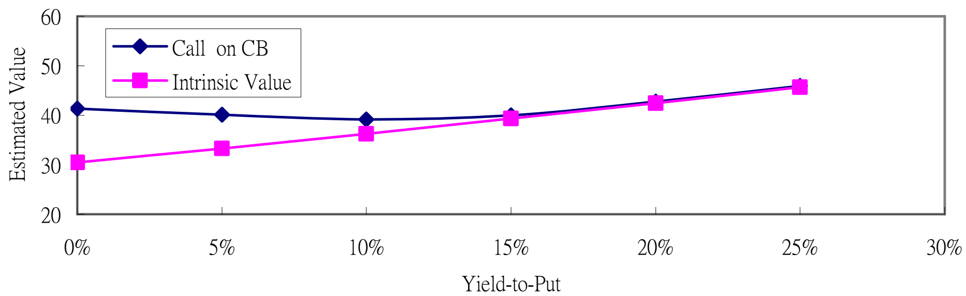
4(d) Effect of Initial Interest Rate



4(e) Effect f Correlation Coefficient



4(f) Effect of Initial Credit Rating



4(g) Effect of Yield-to-Put

Figure 4 The Sensitive Analysis for each parameter

The upper line is the value of Call for various parameters. The lower line is intrinsic value, which is obtained by value difference of CB value and strike price.

The simulation results of the intrinsic value with varying volatility in the stock price are shown in Figure 4(a). From this Figure, we observe that the simulated values of a CB option and the intrinsic value both increase as the volatility of the stock price increases. Since the theoretical CB value is deep in-the-money, the CB option is not much higher than the intrinsic value and these two lines are very close. The time value of a CB option is slightly higher in the case of smaller volatility. The results of the intrinsic value and the volatility of the interest rate are shown in Figure 4(b). Both of the simulated values increase as the volatility of the interest rate increases and the scales of both values increase at an oscillating rate as the volatility increases.

Figure 4(c) shows the simulation results for different initial stock prices. We observe from Figure 4(c) that both simulated values for the CB option and intrinsic value rise as the initial stock price increases. The shape is upward sloping as the initial stock price increases. The value of a CB Option is higher when the initial stock price is low, which indicates that equity investors prefer buying a CB option than holding a CB in the low stock price case.

Figure 4(d) shows the effect of different risk-free interest rates on a CB's option valuation. Since the value of the underlying asset is negatively related to the interest rate, we observe that the CB's option valuation decreases as the risk-free interest rate increases. Most importantly, we can observe that the value of the CB option decreases at a diminishing rate as the interest rate level increases. Compared with the intrinsic value, a CB option is more valuable when a higher interest rate leads to a lower CB value.

Since the processes of stock price and the risk-free interest rate are stochastic, we are curious about whether similar fluctuations have an obvious influence on the simulation results. By changing the number of the correlation coefficient, we show the results through Figure 4(e). We observe that there is a negative relation, but a small influence, for the estimated values with the setting of the correlation coefficient between the stock price and the risk-free interest rate. This is because the interest rate effect as the discount factor will partly counterbalance the stock price effect, even though a higher interest rate should lead to a higher stock price under the settings of a positive correlation coefficient.

In order to reflect the credit risk of a CB, we use corresponding credit spreads of the credit rating in our pricing model. We assume big credit spreads for CBs under investment grade (BBB) in order to reflect a higher possibility of bankruptcy. Figure 4(f) shows the effect of different initial credit ratings on the valuation of a CB option and intrinsic value. In Figure 4(f), we observe that the simulated value dramatically decreases as the credit rating drops. It is important to note that, compared to holding a CB position (i.e., intrinsic value), a CB option is relatively valuable as a credit rating deteriorates, because the option holder can give up the exercise right to call the CB in case of bankruptcy.

Figure 4(g) shows the effect of a put price on the CB's option valuation. It is important to note that a lower put price makes a CB option more attractive than a CB's holding value. In other words, we see the CB option as having a much higher value than intrinsic value in a low put price case.

5. Comments and Conclusions

This paper investigates the deal structure of stripping CBs into their credit component and equity component, i.e., a CB asset Swap and a CB option. We provide pricing models for both the credit component and option component for CB-stripping structured products. We show that a CB asset swap can be priced as an American installment option. Our results indicate that a higher asset swap spread paid by the dealer could lead to an early exercising of the CB option. Comparing to the ad hoc two-stage model used by practitioners, the estimated spread based on the American installment option model should be higher, in order to take into account the early exercise premium.

Based on a Monte Carlo simulation procedure proposed by Longstaff and Schwartz (2001), we can simulate a three-factor CB option model with an early exercise feature. Holding the interest rate and credit rating constant, we find that a CB option is slightly more expensive than holding a CB position, defined as the intrinsic value. Most importantly, our simulation concludes that, compared to holding a CB, a CB call option will be mostly affected by: (1) issuer's credit, (2) interest rate level, and (3) put price.

In a high credit grade and a high put price situation, the valuation of a CB option and intrinsic value should be similar. In other words, a CB option investor should not pay a much higher premium than the intrinsic value. On the other hand, if the CB is considered to be non-investment grade (lower than BBB) and the interest rate (compared to a put yield) is high, then the equity investor should pay a much higher premium for a CB option than the CB intrinsic value.

6. References:

Ait-Sahalia, Y. and A. W. Lo (1998), "Nonparametric Estimation of State-Price Densities Implicit in Financial Asset Prices," *Journal of Finance*, 499-547.

Brady, B. and R. J. Bos (2002), "Record Defaults in 2001 the Result of Poor Credit Quality and a Week Economy," *Standard and Poor's Special Report (Ratings Performance 2001)*, 5-49.

Bloomberg, 1998, "A Derivative for Asia's Season of Financial Discontent," *Bloomberg Magazine*, May.

Brennan, M. J. and E. Schwartz (1977), "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," *Journal of Finance*, 32, 1699-1715.

Brennan, M. J. and E. Schwartz (1980), "Analyzing Convertible Bonds," *Journal of Financial and Quantitative Analysis*, 15, 907-929.

Chance, D.M. and D. Rich (1998), "The Pricing of Equity Swaps and Swaptions," *The Journal of Derivatives*, Summer, 19-31.

Cox, J. C., J. E. Ingresoll and S.A. Ross (1985), "A Theory of the Term Structure of Interest Rates", *Econometrica*, Vol. 36(7), 385-407.

Cox, J.C., S. A. Ross and M. Rubinstein (1979), "Option Pricing: A Simplified Approach", *Journal of Financial Economics*, Vol. 7, 229-264.

Davis, M., W. Schachermayer and R. Tompkins (2001), "Pricing, No-arbitrang Bounds and Robust Hedging of Installment Options," *Quantitative Finance* 1, 597-610.

Deutsche Bank Group, 2001, *Yageo Corp. Stripped Convertible Asset Swap -Indicative Terms and Conditions*.

Duffie, D. and K. Singleton (1999), "Modeling Term Structure of Defaultable Bonds," *Review of Financial Studies*, 12, 687-720.

Euro Money 2002, "The Guide to Global Fixed Income, 2002".

Hull, J. and A. White (1995), "The Impact of Default Risk on the Prices of Options and Other Derivative Securities," *Journal of Banking and Finance*, 19, 299–322.

Hung, M. W. and J. Y. Wang (2001), "Pricing Convertible Bonds Subject to Default Risk," *Journal of Derivatives,* Winter, 75-87.

Ingersoll, J. E. (1977), "A Contingent Claims Valuation of Convertible Securities," *Journal of Financial Economics*, 4, 289-322.

Jarrow, R. A., D. Lando and S. M. Turnbull (1997), "A Markov Model for the Term Structure of Credit Risk Spreads," *Review of Financial Studies,* Vol. 10(2), 481-523.

Jarrow, R. and S. Turnbull (1995), "Pricing Options on Financial Securities Subject to Credit Risk," *Journal of Finance*, 50, 53–85.

Kijima, K. and K. Komoribayashi (1998), “A Markov Chain Model for Valuing Credit Risk Derivatives”, *Journal of Derivatives*, 97-108.

Longstaff, F. A. and E.S. Schwartz (2001), “Valuing American Options by Simulation: A Simple Least-Square Approach,” *The Review of Financial Studies,* Vol. 14(1),113-147.

Longstaff, F. A. and E. S. Schwartz (1995), “Valuing Credit Derivatives,” *Journal of Fixed Income*, 5, 6–12.

MONIS, 2002, “Structuring Convertible Bonds,” MONIS Convertible XL V4.00 User Guide.

Sundaresan, S. M. (2000), “Continuous-Time Methods in Finance: a Review and an Assessment,” *Journal of Finance*, 54, 1569–1622.